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Dependent Scattering by Parallel Fibers: Effects of Multiple Scattering and Wave Interference

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The respective contributions of multiple scattering and wave interference on dependent scattering by a collection of closely-spaced, parallel fibers were investigated in this paper. The former is primarily a near field effect, and the latter occurs in the far field. The present analysis first employs the formulation for the full dependent scattering theory which accounts for both the near- and far-field wave interactions. These effects were then successively removed to derive the scattering properties for different approximations. Expressions for the scattering and extinction cross sections, as well as the general form factor for oblique incidence, were obtained when multiple scattering is neglected. The validity of neglecting multiple scattering is evaluated from both physical and analytical considerations. Neglecting multiple scattering is shown to result in incorrect scattering properties, except for small fiber sizes. The error caused by neglecting multiple scattering decreases when the fibers approach the Rayleigh limit and when the interfiber separation is large.

Nomenclature

a_{jn}	=	coefficient of scattered wave, TE mode
b_{jn}	=	coefficient of scattered wave, TM mode
C	=	cross section
$\mathbf{e}_o$	=	unit vector in the direction of the incident wave
$F(\gamma, \phi)$	=	form factor
H_n	=	Hankel function of the second kind
I_{11}, I_{12}	=	intensity distribution
i	=	$\sqrt{-1}$ or index, 1 to 2
J_n	=	integral order Bessel function
k	=	index, $-\infty$ to ∞
$\mathbf{k}$	=	wave number, $2\pi/\lambda$
k_j	=	complex part of refractive index of fiber j
k_o	=	wave number of vacuum, $2\pi/\lambda_o$
ℓ_o	=	$k_o \cos \phi$
m_j	=	refractive index of fiber j, $n_j - ik_j$
N	=	total number of fibers
n	=	index, $-\infty$ to ∞
n_j	=	real part of refractive index of fiber j
${}^oi_{11}, {}^oi_{12}$	=	intensity functions for independent scattering
$\mathbf{R}$	=	radial vector
r_o	=	radius of fiber
s	=	index, $-\infty$ to ∞
T	=	scattering intensity function
u	=	TM mode scalar potential function
v	=	TE mode scalar potential function
α	=	size parameter, $2\pi r_o/\lambda_o$
γ	=	polar angle
Δ_{mj}	=	total phase lag
δ_{jk}, δ_{ns}	=	Kronecker delta function
ε_j	=	phase shift of the primary incident wave at fiber j relative to the origin
ε_{jm}	=	phase shift of the primary incident wave at fiber j relative to fiber m
θ	=	angle between the line joining the fiber centers and the incident direction
λ	=	wavelength
ϕ	=	angle of incidence

Subscripts

e	=	extinction
j, k	=	index, refers to the fiber
s	=	scattering
w	=	include wave interference effects only

Superscripts

s	=	refers to the scattered wave
o	=	refers to the primary incident wave, or refers to independent scattering
I	=	TM mode
II	=	TE mode

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Introduction

FIBROUS materials are commonly used for thermal insulation in commercial and space systems. The arrangement of fibers in the materials, which refer to the packing density and the fiber orientation, varies with the application. Building insulation and some heat tiles for space systems are, in general, of high porosity and consist of fibers that are loosely packed and randomly oriented. On the other hand, the packing density of fabrics used in re-entry vehicles are usually quite high. Nextel blankets represent a typical example of this type of materials, which are woven from fiber bundles (yarns) each containing a large number of closely packed parallel fibers.

The phenomena of radiative energy transfer through high porosity fibrous materials are relatively simple and can be treated by using the independent scattering assumption. Detailed analyses for the radiative properties and radiative heat transfer models which account for the effect of fiber orientation have been developed.[1-4] On the other hand, radiation analyses for high-density fabric materials are considerably more complicated due to the dependent scattering effects. Dependent scattering occurs when the separations between the scatterers are comparable to the particle size and the wavelength of the incident radiation. Under these conditions, the interaction of radiation with any one particle is affected by the presence of other particles in its vicinity, and hence the term dependent scattering.

Dependent scattering consists of two major phenomena 1) near-field multiple scattering in which radiation scattered by one particle is incident on other particles to be scattered again; and 2) far-field interference of scattered waves by each particle. Both effects must be accounted for in the analysis of radiative properties and radiative transfer of a dense medium. Due to the complexities of these phenomena, few radiative energy transport analyses which account for both effects have been reported in the literature, particularly those for fibrous media.

An approximate method based on the Rayleigh-Debye (also known as Rayleigh-Gans) theory was recently developed for densely packed parallel fibers for the case of perpendicular incidence.[5,6] A detailed discussion on the physical basis of Rayleigh-Debye scattering can be found in Kerker.[7] The fundamental approximation in the Rayleigh-Debye approach is that the phase shift of the electromagnetic (EM) wave in traversing the particle is negligible. For this reason neither the particle size nor the refractive index of the particle can be too large.

The Rayleigh-Debye theory is commonly used for weakly absorbing particles, because the EM wave only suffers negligible reduction of intensity and phase change in traversing the particle. In essence, an EM wave traverses the particle as if there is no particle at all. The near-field multiple scattering effect is then negligible and wave interference in the far-field becomes the dominant effect.

An advantage of the Rayleigh-Debye theory is the simplicity of the resulting formula. By considering only far field wave interference, the scattered intensity is related to that for independent scattering by a "form factor." However, optimal design for high temperature fabrics requires the consideration of utilizing a wide variety of fiber materials and fiber sizes. Since the refractive index of ceramics and most common materials is complex and is usually much different from unity over a wide spectral range, multiple scattering is not negligible except at some wavelengths and for Rayleigh limit fibers. These restrictions impose severe limitations on the application of the Rayleigh-Debye theory in the design of high temperature fabric materials.

Although the applicability of the Rayleigh-Debye theory to practical design of fabric materials is quite limited, there is always the tendency to overextend the utilization of analyses based on this theory. As pointed out by van de Hulst,[8] the simplicity of the Rayleigh-Debye formulae sometimes led to them being applied beyond their range of validity. In order to understand the limitations of the analyses neglecting the near field effect, it is necessary to examine the relative contributions of the near field and the far field effects to dependent scattering.

The purpose of this article is to investigate the effects of near-field multiple scattering and far-field wave interference on the extinction and scattering of radiation by densely packed parallel fibers. The paper will begin with a review of the full dependent scattering theory. Successive approximations to remove the near-field and the far-field effects are applied to reduce the full dependent scattering theory to the Rayleigh-Debye and single scattering formulations, respectively. General formulas for the far-field properties which include only the wave interference effects are derived for Rayleigh limit fibers for oblique incidence. Numerical results will be presented to illustrate the respective contributions of multiple scattering and wave interference to dependent scattering for a specific configuration of parallel fibers.

Theoretical Formulation

In order to evaluate the significance of these two phenomena, the full dependent scattering theory recently developed by Lee[9] for oblique incidence is employed. The analysis accounted for the detailed interactions of EM waves scattered from each fiber, as well as the depolarization of the scattered waves at oblique incidence. Maxwell's equations were solved to obtain the wave functions for the scattered waves. The full dependent scattering formulation is briefly described below in order to provide continuity in the mathematical development.

Full Dependent Scattering Formulation

The formal solution to dependent scattering must be obtained by solving Maxwell's equations. The scalar wave functions u and v corresponding to the z components of the magnetic and electric Hertz vectors, respectively, satisfy the Helmholtz equation

$$\{\nabla^2 + \boldsymbol{k}^2\}\{u, v\} = 0 \tag{1}$$

where $\boldsymbol{k}(= 2\pi/\lambda)$ is the wave number. In the medium containing the fibers, the refractive index is taken to be unity; hence, $\lambda = \lambda_o$ and $\boldsymbol{k} = k_o = 2\pi/\lambda_o$. Within cylinder j, the refractive index is $\boldsymbol{m}_j = n_j - ik_j$, and therefore $\lambda = \lambda_j$, $\boldsymbol{k} = 2\pi/\lambda_j$. The wave functions u and v correspond to the transverse magnetic (TM) and transverse electric (TE) modes, respectively.

The scalar wave functions for each fiber are constructed by accounting for the contributions from all sources. For a system of N parallel fibers as shown in Fig. 1, the total wave functions in the vicinity of a fiber (that is, external to but near a given fiber) consist of the primary incident radiation from an external source and the scattered radiation from all the fibers. The total wave functions can be written as[9]

$$\begin{Bmatrix} u(\boldsymbol{R}) \\ v(\boldsymbol{R}) \end{Bmatrix} = \begin{Bmatrix} u^o(\boldsymbol{R}) \\ v^o(\boldsymbol{R}) \end{Bmatrix} + \sum_{k=1}^{N} \begin{Bmatrix} u_k^s(\boldsymbol{R} - \boldsymbol{R}_k) \\ v_k^s(\boldsymbol{R} - \boldsymbol{R}_k) \end{Bmatrix} \tag{2}$$

where the upper-right superscripts o and s denote the incident and scattered waves, respectively, and $\boldsymbol{R}$ is a radial vector in the cylindrical polar coordinate system attached to the reference frame OXYZ. The subscripts j and k refer to the fibers. The incident and scattered waves are given by

$$\begin{Bmatrix} u_j^o, v_j^o \\ u_j^s, v_j^s \end{Bmatrix} = \sum_{n=-\infty}^{\infty} (-i)^n \exp(in\gamma_{jp}) \cdot \begin{Bmatrix} \varepsilon_j J_n(\ell_o|\boldsymbol{R} - \boldsymbol{R}_j|) \\ -H_n(\ell_o|\boldsymbol{R} - \boldsymbol{R}_j|) * (b_{jn}, a_{jn}) \end{Bmatrix} \tag{3}$$

where $i = \sqrt{-1}$, $\ell_o = k_o \cos\phi$ and ϕ is the angle of incidence, γ_{jp} is the polar angle of a point P in space relative to fiber j, and J_n and H_n are the integral order Bessel function and Hankel function of the second kind, respectively. The unknown wave coefficients a_{jn} and b_{jn} correspond to the transverse electric and transverse magnetic modes of radiation, respectively. The phase shift ε_j of the primary incident wave at the fiber location relative to the origin is given by

$$\varepsilon_j = \exp\{-ik_o\boldsymbol{R}_j \cdot \boldsymbol{e}_o\} \tag{4}$$

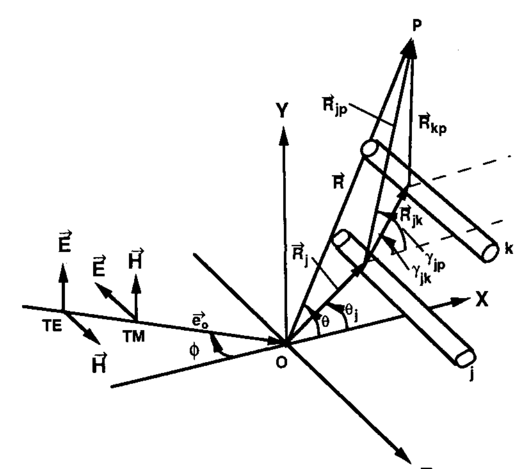

Fig. 1 Plane electromagnetic wave at oblique incidence on a collection of parallel infinite cylinders (ϕ = angle of incidence).

where $\boldsymbol{e}_o$ is a unit vector in the propagating direction of the incident wave. By employing the addition theorem for Bessel functions and the electromagnetic boundary conditions, the following set of equations were obtained for the unknown wave coefficients of an incident TM wave:

$$\sum_{s=-\infty}^{\infty} \sum_{k=1}^{N} \{[\delta_{jk}\delta_{ns} + (1 - \delta_{jk})G_{ks}^{jn\,o}b_{jn}^{\mathrm{I}}]b_{ks}^{\mathrm{I}} + (1 - \delta_{jk})G_{ks}^{jn\,o}b_{jn}^{\mathrm{II}}a_{ks}^{\mathrm{I}}\} = \varepsilon_j\,{}^{o}b_{jn}^{\mathrm{I}} \tag{5}$$

and

$$\sum_{s=-\infty}^{\infty} \sum_{k=1}^{N} \{[\delta_{jk}\delta_{ns} + (1 - \delta_{jk})G_{ks}^{jn\,o}a_{jn}^{\mathrm{II}}]a_{ks}^{\mathrm{I}} + (1 - \delta_{jk})G_{ks}^{jn\,o}a_{jn}^{\mathrm{I}}b_{ks}^{\mathrm{I}}\} = \varepsilon_j\,{}^{o}a_{jn}^{\mathrm{I}} \tag{6}$$

where δ_{jk} and δ_{ns} are the Kronecker delta functions, the superscripts I(TM) and II(TE) denote the mode of the incident wave, and the wave coefficients having the upper-left superscript o refer to those for independent scattering. Expressions for the independent scattering wave coefficients are given in Kerker.[7] The pair-coupling function which relates the influence between a pair of fibers j and k is given by

$$G_{ks}^{jn} = (-i)^{s-n}H_{s-n}(\ell_o|\boldsymbol{R}_j - \boldsymbol{R}_k|)\exp\{i(s - n)\,\gamma_{kj}\} \tag{7}$$

where γ_{kj} is the polar angle that the line joining the centers of fibers j and k makes with the X axis (see Fig. 1). Note that for $j = k$, $(1 - \delta_{jk})G_{ks}^{jn} = 0$. The corresponding equations for a TE mode incident wave can be obtained by replacing $\{b_{jn}^{\mathrm{I}}, a_{jn}^{\mathrm{I}}, {}^{o}b_{jn}^{\mathrm{I}}, {}^{o}a_{jn}^{\mathrm{I}}\}$ by $\{a_{jn}^{\mathrm{II}}, b_{jn}^{\mathrm{II}}, {}^{o}a_{jn}^{\mathrm{II}}, {}^{o}b_{jn}^{\mathrm{II}}\}$ in Eqs. (5) and (6). It is emphasized that Eqs. (5) and (6) follow from the consideration of the general dependent scattering phenomena for an obliquely incident plane wave.

The far field radiation properties of primary interest are the extinction and scattering cross sections and the intensity distribution functions. Since the modulus of the phase shift factor is unity, the radiative properties are independent of the choice of the origin. Hence, the mth cylinder is chosen as the reference origin. By utilizing the asymptotic formula for the Hankel function, the scattering intensity functions are written as[9]

$$\begin{Bmatrix} T_{11}, T_{12} \\ T_{21}, T_{22} \end{Bmatrix} = \sum_{j=1}^{N} \sum_{n=-\infty}^{\infty} e^{in\gamma} \cdot \exp[i\ell_o R_{mj}\cos(\gamma_{mj} - \gamma)] \begin{Bmatrix} b_{jn}^{\mathrm{I}}, a_{jn}^{\mathrm{I}} \\ b_{jn}^{\mathrm{II}}, a_{jn}^{\mathrm{II}} \end{Bmatrix} \tag{8}$$

where $R_{mj} = |\boldsymbol{R}_m - \boldsymbol{R}_j|$, the first and second subscripts in T_{ij} refer to the modes of the incident and scattered waves, respectively, and the dependence of T_{ij} on γ and ϕ has been omitted for brevity. The total scattering cross section for the collection of parallel fibers is obtained by integrating over all the scattered intensities.[7,8] It is given by

$$C_{si}(\phi) = \frac{\lambda}{\pi^2}\int_0^{2\pi} \sum_{m=1}^{2} |T_{im}(\gamma, \phi)|^2\,\mathrm{d}\gamma \tag{9}$$

The extinction cross section is determined from the forward amplitude, i.e., $\gamma = 0$, of the scattered waves, which follows from the extinction theorem[8]

$$C_{ei} = \frac{2\lambda_o}{\pi}\,\mathrm{Re}\left\{\sum_{m=1}^{2} T_{im}(\gamma = 0)\right\} \tag{10}$$

where Re denotes taking only the real part. The subscripts $i = 1$ and 2 in Eq. (9) and (10) refer to the TM and TE mode, respectively. By setting $\gamma = 0$ in the scattering intensity functions of Eq. (8), the TM and TE mode extinction cross sections are given by

$$\begin{Bmatrix} C_{e1} \\ C_{e2} \end{Bmatrix} = \frac{2\lambda}{\pi}\,\mathrm{Re}\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} \exp(i\ell_o R_{mj}\cos\gamma_{mj})\begin{Bmatrix} b_{jn}^{\mathrm{I}} + a_{jn}^{\mathrm{I}} \\ b_{jn}^{\mathrm{II}} + a_{jn}^{\mathrm{II}} \end{Bmatrix} \tag{11}$$

The cross sections for unpolarized incident waves are equal to the average of those for the TM and TE modes. The formulation described above provides the continuity for the analyses in the subsequent sections.

Neglecting Near-Field Interaction

By using the above formulation which accounts for the EM wave interaction in both the near and far fields, the effect of multiple scattering and wave interference can be readily assessed. First, the multiple scattering effect can be examined by removing the near-field EM wave interaction from the full dependent scattering formulation. The resulting formulas, which will be derived for the general case of oblique incidence, then contain only the far-field wave interference effect. The range of validity of the approximation of removing the near-field effect will be discussed later.

To avoid unnecessary repetition of mathematics, it is sufficient to state here that the pair-coupling function G_{ks}^{jn} given by Eq. (7) represents the near-field interaction between any pair of fibers j and k. This is because G_{ks}^{jn} arises from the transformation of the scattered wave from fiber k into secondary incident waves on fiber j. Readers should consult Ref. 9 for additional details. The secondary incident waves on fiber j are discarded by neglecting the terms involving G_{ks}^{jn} which is equivalent to removing the near-field multiple scattering effect.

The unknown wave coefficients for this case reduce directly from Eqs. (5) and (6) as

$$\{b_{jn}, a_{jn}\} = \varepsilon_{jm}\{{}^{o}b_{jn}, {}^{o}a_{jn}\} \tag{12}$$

where $\varepsilon_{jm} = \exp\{-ik_o(\boldsymbol{R}_j - \boldsymbol{R}_m)\cdot\boldsymbol{e}_o\} = \exp\{-ik_oR_{mj}\cos\gamma_{mj}\}$. The superscripts I and II denoting the TM and TE modes have been omitted for brevity. Consequently, the scattering intensity functions become

$$\{T_{11}(\gamma), T_{12}(\gamma)\} = \sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} \varepsilon_{jm} \cdot \exp\{i\ell_oR_{mj}\cos(\gamma_{mj} - \gamma)\}\cdot e^{in\gamma}\{{}^{o}b_{jn}^{\mathrm{I}}, {}^{o}a_{jn}^{\mathrm{I}}\} \tag{13}$$

$$= \sum_{j=1}^{N}\exp\{i\Delta_{mj}\}\sum_{n=-\infty}^{\infty} e^{in\gamma}\{{}^{o}b_{jn}^{\mathrm{I}}, {}^{o}a_{jn}^{\mathrm{I}}\} \tag{14}$$

where $\Delta_{mj} = \ell_oR_{mj}\{\cos(\gamma_{mj} - \gamma) - \cos\gamma_{mj}\}$ is the total phase lag. The dependence of T_{11} and T_{12} on the incident angle ϕ has been omitted for brevity. The corresponding functions T_{21} and T_{22} are defined similarly by replacing $\{{}^{o}b_{jn}^{\mathrm{I}}, {}^{o}a_{jn}^{\mathrm{I}}\}$ by $\{{}^{o}b_{jn}^{\mathrm{II}}, {}^{o}a_{jn}^{\mathrm{II}}\}$ in the above equation. The total phase lag arises from two sources. The part due to the relative location of fiber j with respect to the incident wave is denoted by ε_{jm}. The second part is due to the relative location of fiber j to the point of observation.

Form Factor

The far-field scattering intensity distributions are proportional to the square of the absolute magnitudes of the intensity functions. The scattering intensity functions can be further simplified if either one of the following assumptions is made 1) all the fibers are of the same size, or 2) the fibers are in the Rayleigh limit, i.e., $\alpha(=k_or_o) \ll 1$, where α is the size parameter and r_o is the fiber radius. The first assumption is automatically satisfied for Rayleigh limit fibers. By applying either one of these assumptions, the intensity distributions become

$$\{I_{11}(\gamma, \phi), I_{12}(\gamma, \phi)\} = \frac{2}{k_o \pi R} \cdot NF(\gamma, \phi)\, \{{}^o i_{11}(\gamma, \phi), {}^o i_{12}(\gamma, \phi)\} \quad (15)$$

where the general form factor for oblique incidence $F(\gamma, \phi)$ is defined as

$$F(\gamma, \phi) = \frac{1}{N^2} \left| \sum_{j=1}^{N} \exp\{i\Delta_{mj}\} \right|^2 = \frac{1}{N^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \cos(\Delta_{mj} - \Delta_{mk}) \quad (16)$$

The upper-left superscript o associated with ${}^o i(\gamma, \phi)$ denotes independent scattering. By setting $\phi = 0$ deg for perpendicular incidence, the form factor given by Eq. (16) reduces to that of Eq. (2) in White and Kumar.[5]

It is obvious that the form factors for various fiber arrangements given in Ref. 5 for perpendicular incidence can be converted to those for oblique incidence by simply replacing k_o with ℓ_o in all the equations involving the form factor. Based on this simple extrapolation, the form factor for randomly oriented parallel fibers having the same size can be deduced as

$$F(\gamma, \phi) = 1 - 4f_v \left\{ a + b\left(4\alpha \cos\phi \sin\frac{\gamma}{2}\right)^2 \right\} \quad (17)$$

where f_v is the volume fraction, and $a = 1$, $b = -\frac{1}{8}$ for moderate fiber concentration and $a = 0.1592$ and $b = 0.0852$ for higher concentrations.[5]

Scattering Cross Section

The scattering cross section is obtained by integrating the scattered intensity distribution over all angles as defined by Eq. (9). For fibers with size comparable to the wavelength, the expression cannot be further simplified and must be evaluated numerically. For Rayleigh limit fibers the asymptotic expansions for the wave coefficients obtained by Wait[10] can be used to obtain explicit expressions for the total scattering cross section at oblique incidence. By neglecting terms of order higher than α^2, the following expressions are obtained:

$$C_{sw,1}(\phi) = N \int_0^{2\pi} F(\gamma, \phi) \left(\frac{\pi\alpha^4}{8k_o}\right) \cdot \Bigg[\Bigg\{ \cos^3\phi \,\mathrm{Im}\,(m^2 - 1) + 2\cos\phi \sin^2\phi \cos\gamma \,\mathrm{Re}\left(\frac{m^2-1}{m^2+1}\right) \Bigg\}^2 + \Bigg\{ \cos^3\phi \,\mathrm{Re}(m^2 - 1) - 2\cos\phi \sin^2\phi \cos\gamma \cdot \mathrm{Im}\left(\frac{m^2-1}{m^2-1}\right) \Bigg\}^2 \Bigg] d\gamma \quad (18)$$

for the TM mode, and

$$C_{sw,2}(\phi) = N \int_0^{2\pi} F(\gamma, \phi) \left(\frac{\pi\alpha^4}{2k_o}\right) \left| \left(\frac{m^2-1}{m^2+1}\right) \right|^2 \cdot \cos^2\phi\, (\sin^2\phi + \cos^2\gamma)\, d\gamma \quad (19)$$

for the TE mode, where Re and Im refer to the real and imaginary parts, respectively, and the subscript w denotes that only wave interference effects were accounted for. The subscript j associated with the fiber refractive index has been suppressed because the fibers are identical. The terms involving $\sin^2\phi$ arise from the cross polarization modes. The cross section for unpolarized radiation is simply the average of the two components.

The Rayleigh limit total scattering cross section at oblique incidence for randomly oriented parallel fibers can be derived by substituting Eq. (17) into Eqs. (18) and (19). The integration can be carried out to give

$$C_{sw,1} = N\left(\frac{\pi^2\alpha^4}{4k_o}\right) \Bigg\{ [1 - 4f_v(a + 8b\alpha^2\cos^2\phi)] \cdot \left[\cos^6\phi |m^2 - 1|^2 + 2\cos^2\phi \sin^4\phi \left|\frac{m^2-1}{m^2+1}\right|^2 \right] + 64b\alpha^2 f_v \cos^6\phi \sin^2\phi * \left[\mathrm{Re}(m^2 - 1)\,\mathrm{Im}\left(\frac{m^2-1}{m^2+1}\right) - \mathrm{Im}(m^2 - 1)\,\mathrm{Re}\left(\frac{m^2-1}{m^2+1}\right) \right] \Bigg\} \quad (20)$$

for the TM mode, and

$$C_{sw,2} = N\left(\frac{\pi^2\alpha^4}{2k_o}\right) [1 - 4f_v(a + 8b\alpha^2\cos^2\phi)] \cdot \cos^2\phi(1 + 2\sin^2\phi) \left|\frac{m^2-1}{m^2+1}\right|^2 \quad (21)$$

for the TE mode. The total scattering cross section for other fiber arrangements can be obtained similarly from Eqs. (18) and (19) by using the appropriate form factor. If a, b, and ϕ are set to zero, Eqs. (20) and (21) reduce to the well established independent scattering cross section at normal incidence.

Extinction Cross Section

The extinction cross section can be obtained from the forward intensity as indicated by the extinction theorem in Eq. (10). By setting γ to zero in Eq. (13), the scattering intensity functions become identical to those for independent scattering. Consequently, the extinction cross section is given by

$$C_{ew,1}(\phi) = \frac{2\lambda_o}{\pi} \mathrm{Re} \sum_{j=1}^{N} \sum_{n=-\infty}^{\infty} \{{}^o b_{jn}^{I} + {}^o a_{jn}^{I}\} = \sum_{j=1}^{N} {}^o C_{e1}(\phi) \quad (22)$$

where ${}^o C_{e1}(\phi)$ is the single fiber extinction cross section and the subscript 1 refers to the TM mode. The TE mode cross section can be written similarly.

Equation (22) reveals that if multiple scattering is neglected, the total extinction cross section simply equals to the sum of those for the individual fibers. This result should not be a surprise because discarding the near-field multiple scattering is equivalent to assuming that the scattered waves from a fiber traverse other fibers with negligible change in the phase and amplitude. It should be emphasized that this assumption is adequate only for fibers satisfying the Rayleigh-Debye restrictions. Because the forward amplitudes of the scattered waves from Rayleigh fibers are not modified, the total extinction cross section is equal to the sum of that for a single fiber, since it is determined from the forward amplitude only.

Neglecting Far-Field Interaction

The next step in the theoretical consideration is by neglecting the wave interference effect in the far field from the above formulation. Wave interference occurs when there is a definite phase relationship between the scattered waves from fibers in the medium. Such relations arise from the relative locations of the fibers with respect to each other and to the incident wave. The lowest order approximation of dependent scattering is obtained by neglecting the presence of the other scatterers altogether. Thus the scattered waves from all scatterers are treated as uncorrelated. The total wave potential in the vicinity of a fiber then consists of contributions due only to the external incident wave and the scattered wave from the fiber. Consequently, the total wave potentials reduce to the well-established result for single scattering.[7,8]

The independent scattering results can also be obtained by setting the phase lags in Eq. (16) to be equal, i.e., $\Delta_{mj} = \Delta_{mk}$. For nonpenetrating fibers this condition is admitable only if the indices j and k refer to the same fiber, i.e., there is only one fiber in the medium. The form factor then reduces to 1. Equations (18) and (19) can be integrated to give the independent scattering cross sections.

Physical Assessment of Near-Field Effect

The form factor and scattering cross sections have been derived by neglecting the multiple scattering effect. The only other assumption made had been that the fibers should be either of the same size (not limited to Rayleigh limit fibers) or in the Rayleigh limit. Although identical formulas are obtained for either assumption, their range of validity must be evaluated. In order to assess the accuracy of these formulas, it is necessary to examine the influence of multiple scattering on the polarization of the scattered waves.

Based on the consideration of independent scattering, the scattered wave from a fiber propagates along the surface of a cone whose half apex angle is defined by the angle between the incident direction and the fiber axis.[7,8] The polarization of the scattered wave does not change for perpendicular incidence, whereas it decomposes into both the TM and TE modes at oblique incidence. At oblique incidence decomposition of the polarization of the scattered wave for a single mode incident wave gives rise to secondary incident waves of both TM and TE modes.

For a primary incident TM wave, the total TM wave potential in the vicinity of a fiber consists of three sources 1) the primary incident wave (from an external source); 2) the TM mode scattered wave from the fiber; and 3) the secondary TM mode incident waves due to the successive scattering and depolarization of both TM and TE mode waves by fibers in the medium. The total TE mode wave potential, on the other hand, contains only two sources 1) the TE mode scattered waves from the fiber; and 2) the secondary TE mode incident waves arising from the depolarization of TM mode waves. The TE mode waves arise entirely from the depolarization of the TM mode incident waves.

Delineation of the various contributors to the total wave potentials underlines the significance of the near-field multiple scattering effect. Since the secondary incident waves originate from this near-field effect, neglecting multiple scattering amounts to discarding all the secondary radiation sources due to the decomposed components of polarization. Furthermore, multiple scattering modifies the amplitude and phase of the scattered waves due to absorption and scattering which are inherently affected by the size and optical properties of the fibers.

For fibers that are much smaller than the wavelength, i.e., Rayleigh scatterers, the fibers act as point scatterers and their extinction cross sections are small. Since scattering results from inhomogeneities such as scatterers or local density fluctuations, the disturbance to an EM wave traversing the medium would be small if the refractive index of the fibers is close to that of the medium. The conditions of point scatterers and small deviation of refractive index from that of the medium ensure that both the attenuation and phase shift of the EM wave in traversing the medium are negligible. Under these conditions, the secondary incident waves resulting from successive scattering by the scatterers would necessarily be small. These are exactly the conditions for the Rayleigh-Debye theory.

On the other hand, if the fiber sizes are comparable to the wavelength and their refractive index is much different from that of the medium, the disturbance to the EM wave traversing the medium would be substantial. The polarization, amplitude, and distribution of the scattered waves will be greatly affected by the properties of the fibers. In a medium of closely packed fibers, the scattered waves from a fiber will interact with and be modified by other fibers. Hence, the influence of each fiber on the successive scattering of radiation is significant. In general, the multiple scattering effect is not negligible.

Numerical Results and Discussion

The successive analytical development presented above provides a basis for investigating the roles of multiple scattering and wave interference on dependent scattering by a dense medium of parallel fibers. Since neglecting multiple scattering is equivalent to considering wave interference only, the difference between the properties based on the full dependent scattering theory and those based on wave interference then reveals the effect of multiple scattering. The difference between the properties based on wave interference and those for single scattering then shows the effect of wave interference.

In order to demonstrate the respective effects of wave interaction in the near-field and far-field, numerical results are being presented for a row of five identical fibers spaced equally apart. The plane containing the axis of the fibers makes an angle θ with the propagating direction of the incident wave as depicted in Fig. 2. The interfiber clearance is c and the fiber diameter d is 1 μm. For the purpose of illustration, two wavelengths, 0.32 and 20.66 μm, are considered and the optical properties of glass[11] are used. The refractive index is $\boldsymbol{m} = 1.546$ at 0.32 μm and $\boldsymbol{m} = 1.65-1.28i$ at 20.66 μm. The size parameter α is 9.82 and 0.152 for the two wavelengths, respectively, with the smaller α being roughly in the Rayleigh limit.

Numerical results are calculated for $c/d = 0.1$ and 1.0–5.0 at 1.0 increment, and $\theta = 0$, 30, 60, and 90 deg. They include the extinction and scattering cross sections per fiber for the three analyses 1) full dependent scattering theory (FDS); 2) wave interference effect (WI) only, i.e., neglecting multiple scattering; and 3) single scattering (SS), i.e., neglecting both multiple scattering and wave interference effects. In addition, only results for normal incidence are shown, because the results at oblique incidence reveal trends similar to those for normal incidence, except their magnitudes are smaller. It is reiterated that the extinction cross section with only the wave interference effect is equal to that for SS because the forward

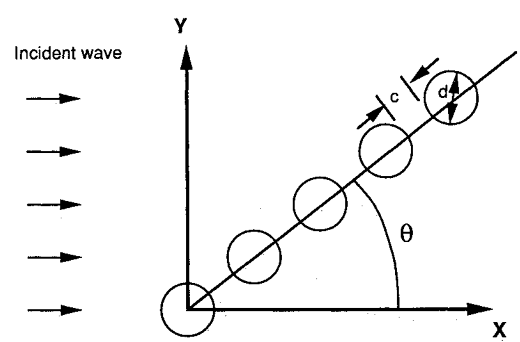

Fig. 2 Configuration of five equally spaced parallel fibers.

scattering amplitudes for the two cases are identical, as shown in the fomulation.

Figure 3 shows the variation of the radiative cross sections with fiber orientation at 0.32 μm (α = 9.82) for c/d = 0.1 and 5. The results for this wavelength are particularly illustrative of the error when the near-field multiple scattering effect is neglected. This is because at this wavelength the refractive index is real and the fibers are purely scattering, so that the extinction and scattering cross sections are equal, as are shown for the FDS and single scattering results. The WI results are, however, entirely ***wrong*** because the scattering cross section is not equal to the extinction cross section.

The deviations of the WI results from those of FDS and single scattering reveal the effects of multiple scattering and wave interference, respectively. If the fibers are aligned in the propagating direction of the incident wave, the forward amplitudes of the scattered waves from each fiber are reduced due to successive scattering by each fiber. Consequently, the extinction cross section is lowest for θ = 0 deg and highest for θ = 90 deg. Since the multiple scattering effect increases with decreasing fiber separation, the forward scattered amplitude is attenuated more at smaller fiber separation. The extinction cross section, which is determined by the magnitude of the forward scattering amplitude, is therefore lower for small fiber separations. The opposite trend is predicted if multiple scattering is neglected. The WI scattering cross section fortuitously approaches that of the extinction cross section when the fibers are aligned normal to the incident wave, due to reduced wave interference for this fiber arrangement.

Figures 4 and 5 show the variation of the cross sections with fiber separation at 0.32 μm for θ = 0 and 90 deg, respectively. For fibers aligned at θ = 0 deg, WI predicts the physically incorrect result of a greater scattering than extinction cross section, as shown in Fig. 4. When the fibers are aligned normal to the incident wave, the WI scattering cross section deviates less from the extinction cross section as the interfiber separation increases and multiple scattering becomes smaller. The error is still noticeable at c/d = 5 due to the neglect of near-field multiple scattering.

Figures 6–9 show the scattering properties at 20.66 μm (α = 0.152). The refractive index is complex and hence the fibers are both absorbing and scattering. Figures 6 and 7 depict the variations of the radiative cross sections with fiber orientation for c/d = 0.1 and 5, respectively. Because of the long wave-

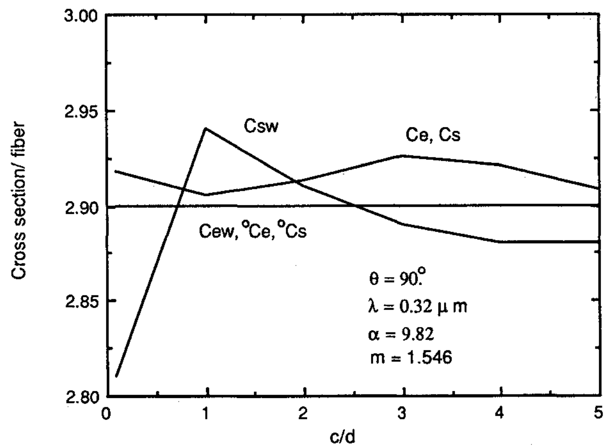

Fig. 5 Extinction and scattering cross sections for fibers aligned normal to the incident wave (d = 1.0 μm).

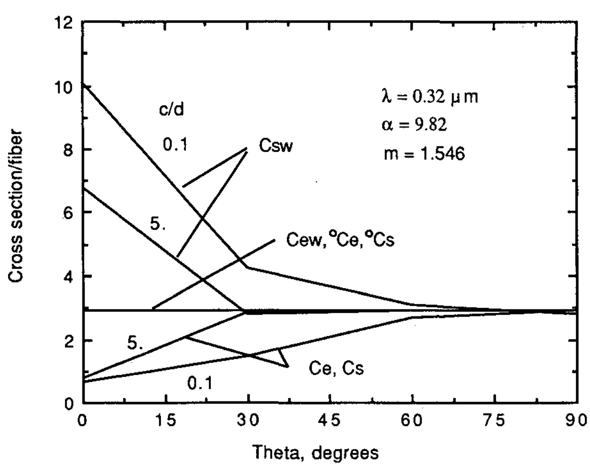

Fig. 3 Variation of the extinction and scattering cross sections with fiber orientation for normal incidence (d = 1.0 μm).

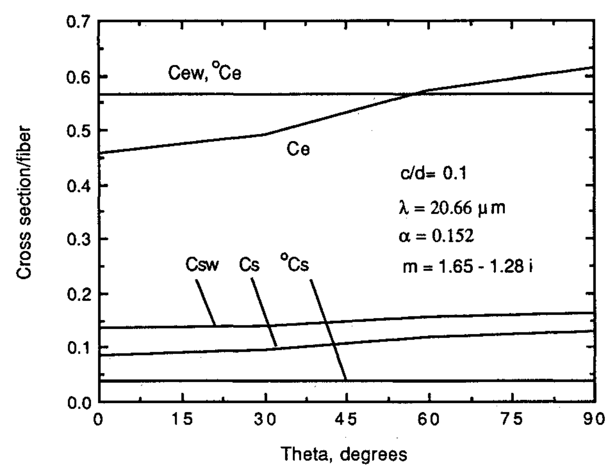

Fig. 6 Extinction and scattering cross sections for normal incidence at c/d = 0.1 (d = 1.0 μm).

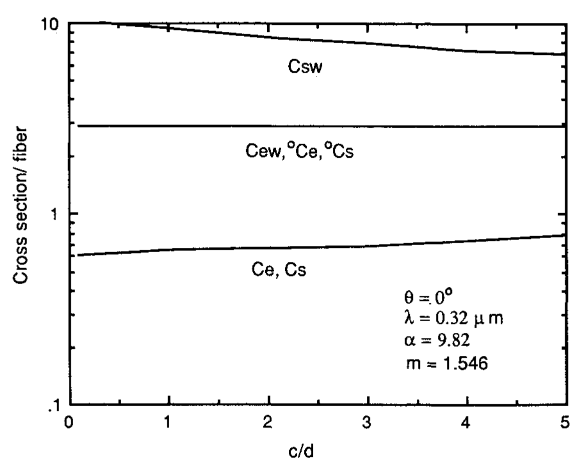

Fig. 4 Extinction and scattering cross sections for fibers aligned in the direction of the incident wave (d = 1.0 μm).

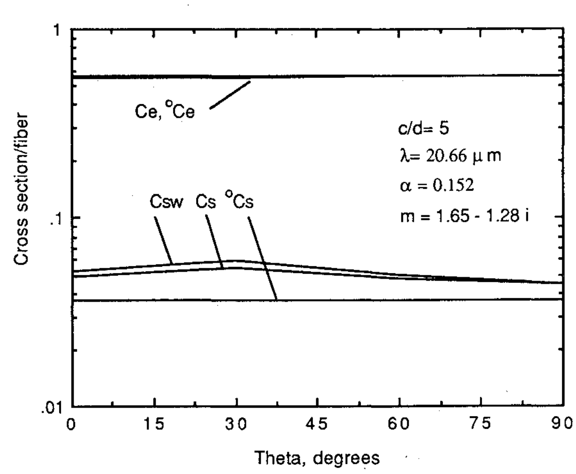

Fig. 7 Extinction and scattering cross sections for normal incidence at c/d = 5 (d = 1.0 μm).

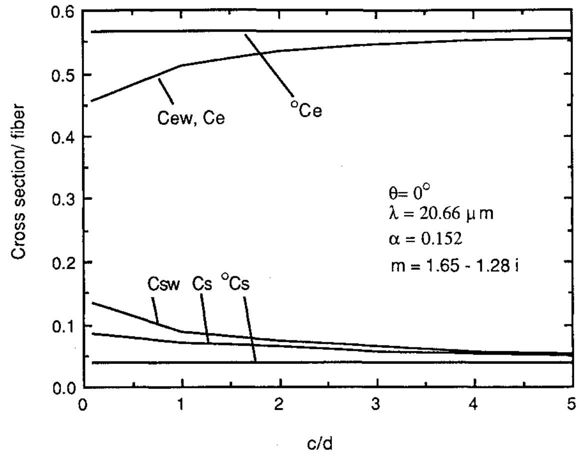

Fig. 8 Variation of the extinction and scattering cross sections with fiber separation for fibers aligned in the direction of the incident wave (d = 1.0 μm).

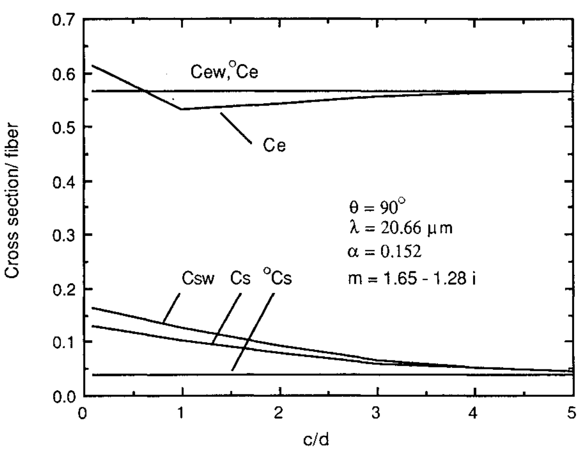

Fig. 9 Variation of the extinction and scattering cross sections with fiber separation for fibers normal to the incident wave (d = 1.0 μm).

length (20.66 μm), multiple scattering is significant even for $c/d = 5$, as shown by the deviations of the WI scattering cross section from the FDS and SS results (Fig. 7).

Figures 8 and 9 show the variation of the extinction and scattering cross sections with fiber separation for $\theta = 0$ and 90 deg, respectively. These results show that both multiple scattering and wave interference are more significant for small fiber separation. The extinction and scattering cross sections approach those for single scattering as the fiber separation increases, due to the diminishing influence of the near-field and far-field effects. These results illustrate that the discrepancy between the WI and FDS results diminishes as the fiber size parameter decreases since the Rayleigh-Debye conditions are progressively approached. Analyses neglecting multiple scattering then become a reasonable approximation to the FDS theory.

Conclusion

Analysis of radiative energy transport through dense media requires the application of radiative properties which account for the dependent scattering effects.[12] Dependent scattering includes EM wave interaction in the near-field and far-field. In the near-field multiple scattering is dominant except under the restrictive conditions of the Rayleigh-Debye theory which only accounts for the wave interference in the far field. In this paper the full dependent scattering theory recently developed by Lee[9] was employed to derive the scattering properties of closely spaced parallel fibers by successively discarding the near-field and far-field effects.

By discarding multiple scattering, the form factor for oblique incidence for arbitrary fiber size was obtained. The application of this form factor is valid only for Rayleigh limit fibers. This form factor was reduced to the well known formula of the Rayleigh-Debye theory at perpendicular incidence. The single scattering results are obtained when the wave interference effect is further neglected. The similarity of the form factor for general fiber sizes and for small fibers underlines the concern of van de Hulst[8] that the simplicity of the form factors sometimes led them to be applied beyond their range of validity.

The successive development of the scattering properties for the different approximations allowed the respective roles of multiple scattering and wave interference to be investigated. Multiple scattering gives rise to cross mode secondary incident radiation sources. It also modifies the forward amplitude of the scattered waves which affects the extinction cross section of the fibers. Discarding the near-field interactions can result in significant errors in the prediction of the extinction and scattering cross sections of the fibers. Only under very restrictive conditions of the Rayleigh-Debye theory is multiple scattering negligible.

The importance of multiple scattering was demonstrated by considering the scattering of EM waves by a row of parallel fibers. Numerical results are shown for different orientations of the fibers with respect to the incident wave and for different interfiber spacings. The results show clearly the limitations of the analyses neglecting multiple scattering. Hence, application of the form factors to treat dependent scattering must be exercised with caution.

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